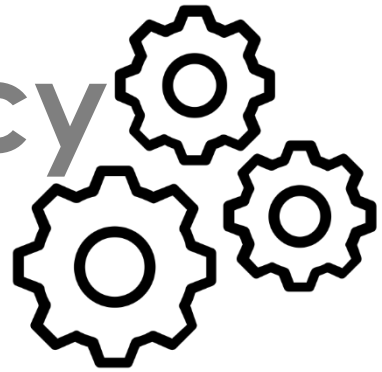
Lingfield Education Trust

Calculation Policy



June 2023

Contents

Background, Purpose and Aims

Mathematics is a subject in which its learning episodes can be taught in multiple different ways, using multiple different representations and methods; this can cause significant confusion and cognitive overload for some students, especially lower attaining students.

The purpose of this document is to provide teachers and staff, who support students in mathematics lessons at Lingfield Education Trust, with an easy-reference guide to the methods that could be employed in the teaching of mathematics.

The key principles underpinning this policy are:

- The importance of mental calculation methods, that are themselves built on secure factual knowledge.
- Giving pupils in each year group a reliable method for calculating that they can apply to varied representations, reasoning and problem-solving. Although there is minimal reference to bar modelling and part-part-whole models in the document, pupils should still be exposed to them regularly through your maths curriculum – this document is for the strategies you would use to complete the missing numbers in both of the aforementioned models.
- Reducing the amount of variation pupils are exposed to in the initial learning phase of calculating in a given year group. Variation is essential to a deep understanding, however we understand that a firm foundation is needed first.
- The importance of the concrete, pictorial and abstract phases of learning.
- Using the right manipulative at the right time – if it is needed.
- Building on prior learning through the careful sequencing of strategies.

The aim of this document is to allow staff to synchronise their practise, to ensure students encounter the same methods throughout their mathematical journey, regardless of their teacher. The aim is that this will provide consistency for students in the long-term and therefore aid in improving their outcomes.

This document was created by members of Lingfield Education Trust's Maths Network based on their teaching expertise, the most up-to-date research and through the study of effective exemplars.

Concrete, Pictorial and Abstract

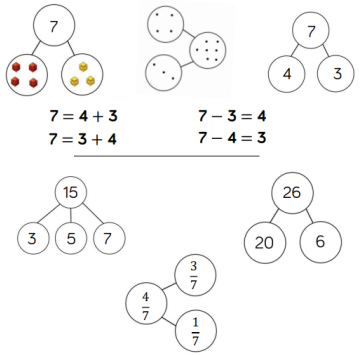
Throughout this document each approach is split into three stages: concrete, pictorial and abstract. The idea is through a systematic approach students will begin, where possible, to explore mathematics by using physical manipulatives so that at the end of the process students should be able to form their own generalisations of mathematical rules.

Concrete	During the concrete stage, pupils will have the opportunity to work with manipulatives and other physical objects in order to understand the mathematical concept. There will be times where this is not possible or effective; in these cases students should begin at pictorial stage.
Pictorial	During the pictorial stage, pupils should be able to pictorially or diagrammatically represent ideas discovered during the Physical Stage. Again there may be occasions where this is not effective and so pupils should start at the abstract stage.
Abstract	During the abstract stage, pupils should no longer require a diagram to understand the concept. They should have formed comprehensive generalisations during which the underlying mathematics is fully understood.

Models & Structures

This document aims to outline the main calculation strategies to be used progressively across school. There are however a range of models and representations that help pupils draw out the structure of the maths behind a task/question – in other words help pupils identify the operation and arithmetic required. This page details some of the most effective that you should use to help pupils expose the structure of the maths before they apply a mental or written strategy to complete the calculation(s).

Part-Part-Whole Model



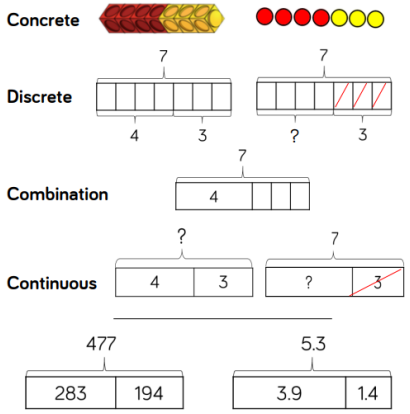
This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part. Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (Single)

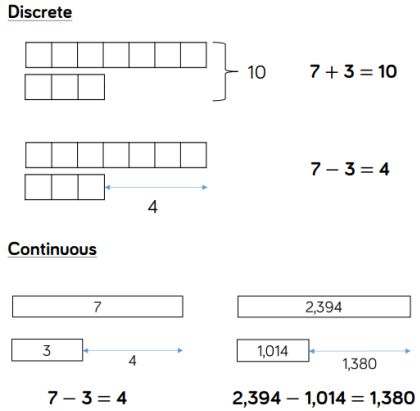


The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure. Cubes and counters can be used in a line as a concrete representation of the bar model. Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model. Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (Multiple)



The multiple bar model is a good way to compare quantities whilst still unpicking the structure. Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Factual Knowledge

The written calculation strategies contained in this document are built up on the mental calculation strategies outlined on the next page, however these themselves are built on secure factual knowledge (fact fluency). These are the key milestones in what **factual knowledge** pupils should know to automaticity and by when. This does not just mean rote learning but using strategies to develop understanding through to automaticity. Programs to use for this are one of: Number Sense, NCETM Mastering Number or WR Fluency Bee.

EYFS

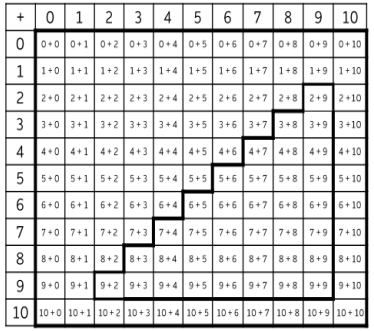
Have a deep understanding of number to 10; including the composition of each number.

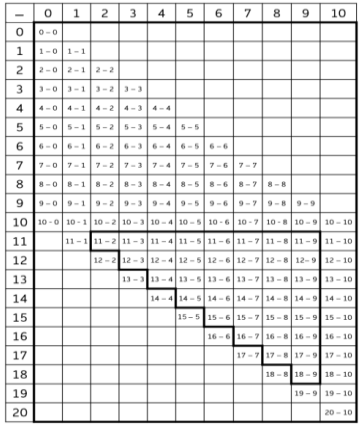
Subitise to 5.

Automatically recall (without reference to rhymes or other aides) number bonds to 5 (including subtraction facts) and some to 10 including doubles.

Year 1

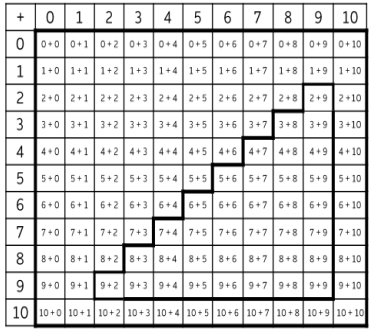
All addition and subtraction facts to 20 that do not bridge 10. They will be exposed to bridging 10 but automaticity is not required.

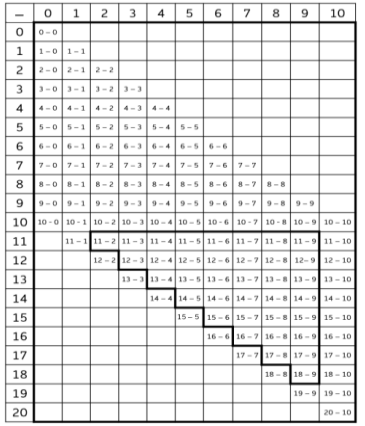




Year 2

All addition and subtraction facts to 20 including those that bridge 10.





Year 4

All multiplication and division facts to 12 x 12.

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Mental Calculation Expectations

	Addition	Subtraction	Multiplication	Division
YR	• Perceptually subitise to 10 • Conceptually subitise to 5 • Find the total number of items in two groups, up to a total of 10 *(combine and subitise, count all (aggregation), use known facts)* • 1 more to 10 • Add zero, within numbers to 10	• 1 less to 10 • Remove from a small group and find how many are left, up to a total of 10 *(take away and subitise, take away and count how many are left, use known facts)* • Subtract zero to 10	• Doubles to 5	
Year 1	• Subitising 1-5 • Recognizing numbers on tens frames • Add 1-digit to tens • Add 1-digit to teens • Number Bonds to 10 • Bridging 10 single digits • Near doubles to 5, e.g. 3+2	• Subtract pairs of 1-digit numbers • Subtraction facts to 10 • Bridging 10 by single digit subtraction • Subtract1-digit from teens • Subtract1-digit from ten •	• Double numbers to 5 • Count forwards and backwards in 2s, 5s and 10s	• Halve even numbers to 10
Year 2	• Bridging 10 (TU + U) • 1-digit to a multiple of ten (e.g. 60 + 5) • Add multiples of 10 to a 2-digit number (e.g. 27 + 60) • Add three 1-digit numbers • Number Bonds to 20 • Number Bonds to 100 in 10s • Add 10 to 2-digit numbers using place value • Add 11 by adding 10 add 1 • Add 9 by add 10 take 1 • Near doubles to 10, e.g 6+5	• Subtract 10 from a 2-digit number using place value • Bridging any 2-digit10 by single digit subtraction • Subtract 1-digit from multiple of 10 • Subtraction facts to 20 • Subtraction facts to 100 in 10s • Subtract 11 by subtracting 10 then 1 • Subtract 9 by subtracting 10 and adding 1	• Double numbers to 10 • Double any multiple of 10 up to 50 • Recognize odd and even • Rapid recall of x2,10,5 as a minimum	• Halve even numbers to 20 • Halve any multiple of 10 with an even tens digit up to 100 • Rapid recall of division facts for x2,10,5 as a minimum
Year 3	• Add 100 to any 3-digit number using place value • Bridging to 3-digit • Add pairs of multiples of 10 up to 2-digit using bonds • 2-digit Near Doubles (teens and tens, e.g. 14 + 13, 30 + 20) • 2-digit near 10s round up (e.g. 27 + 19/21) • Add any 2-digit numbers using partitioning • Add any 2-digit numbers using counting on	• Subtract 100 from any 3-digit number using place value • Bridging HTU by U subtraction • Subtract a 2-digit number from a multiple of 10 • Subtract pairs of multiples of 10 up to 2-digit using bonds • Subtract near multiples of 10 rounding up • Subtract pairs of 2-digit using partitioning • Subtract pairs of 2-digit using counting on	• Double any multiple of 10 up to 100 • Find 4 of a number by doubling and doubling again • Rapid recall of x3, 4,8 as a minimum • Multiply any 2-digit number by 10 • Multiply TU x U using partitioning • Use place value and known facts to TU x U, e.g. 80 x 3	• Halve any multiple of 10 up to 100 • Find a quarter by halving and halving again • Rapid recall of division facts for x3,4,8 as a minimum • Identify the remainder when dividing TU by 2,10,5 • Divide any 3-digit multiple of 10 by 10 • Use place value and known facts to HTU ÷ U, e.g. 400 ÷ 8

Experience has shown us that longer, more complex written methods often go wrong through the **mental calculations** within them.

It is essential that pupils are taught these mental calculation skills.

Once pupils have mastered the relevant mental and written methods for their year group, it is advisable for them to **reason about which method suits a given calculation – what was the most efficient way of doing it!**

Please see our mental calculation policy for further detail to support these expectations.

Mental Calculation Expectations

Year 4	• Add 1000 to any 4-digit number using place value • Bridging up to 4-digit • Add pairs of multiples of 10 up to 3-digit using bonds • 2-digit Near Doubles to 50, e.g. 36 + 37 • 2-digit near 10s round up & down (e.g. 27 + 19/21) • Add any 3-digit numbers using partitioning • Add any 3-digit numbers using counting on	• Subtract 1000 from any 4-digit number using place value • Bridging THTU by U subtraction • Subtract pairs of multiples of 10 up to 3-digit using bonds • Subtract near multiples of 10 rounding up and down • Subtract any 3-digit numbers using partitioning • Subtract any 3-digit numbers using counting on •	• Double any 2-digit number • Double any multiple of 100 • Rapid recall of all tables to 12x12 • Multiply three 1-digit numbers • Multiply any number to 100 by 10/100 • Multiply HTU x U using partitioning • Use place value and known facts to HTU x U, e.g. 400 x 3	• Halve any even number to 100 • Rapid recall of all division facts for tables to 12x12 • Identify the remainder when dividing HTU by 2,10,5 • Divide any number to 1000 by 10/100 • Use place value and known facts to THTU ÷ U, e.g. 1200 ÷ 3
Year 5	• Use place value to add powers of 10 to 1,000,000 • Bridging (U.t + .t) • 2-digit Near Doubles to 100, e.g. 76 + 77 • Add near hundreds (e.g. 427 + 198) • Add any U.t pairs (e.g 3.5 + 2.8) using partitioning • Add any U.t pairs (e.g 3.5 + 2.8) using counting on • Add pairs of multiples of U.t by making x10 larger	• Use place value to subtract powers of 10 up to 1,000,000 • Bridging U.t by U subtraction • Subtract near hundreds (e.g. 427 - 198) • subtract any U.t pairs (e.g 3.5 - 2.2) using partitioning • subtract any U.t pairs (e.g 3.5 - 2.7) using counting on • Subtract pairs of multiples of U.t by making x10 larger	• Double 3-digit multiples of 10 • Double U.t • Multiply whole numbers by 10,100,1000 • Multiply U.t using partitioning • Use place value and known facts to THTU x U, e.g. 8000 x 3 • Multiply pairs of multiples of 10 with same place value ,e.g. 400 x 300 • Multiply by 50 by multiplying by 100 and halving • Multiply by 25 by multiplying by 100 and halving and halving again • Multiply by 20 by multiplying by 10 and doubling • Multiply by 5 by multiplying by 10 and halving	• Halve 3-digit multiples of 10 • Halve any whole number • Find the remainder when dividing TU by any single digit • Divide whole numbers by 10,100,1000 • Use place value and known facts to TTHTU ÷ U, e.g. 64000 ÷ 8 • Multiply pairs of multiples of 10 with same place value, e.g. 800 ÷ 200
Year 6	• Use place value to add powers of 10 to any number • Bridging (U.th + .th) • Near doubles to tenths (e.g. 1.7 + 1.6) • Near tens to tenths (e.g. 4.2 + 1.9) • Add any U.th pairs (e.g 3.52 + 2.87) using partitioning • Add any U.th pairs (e.g 3.52 + 2.87) counting on	• Use place value to subtract powers of 10 from any number • Subtract using near tens to tenths, e.g. 4.6 – 1.9 • Subtract any U.th pairs (e.g 3.52 - 2.31) using partitioning • Subtract any U.th pairs (e.g 3.52 - 2.31) using counting on	• Double any number including to 2dp • Multiply whole numbers and decimals by 10,100,1000 • Multiply U.th x U using partitioning • Use place value and known facts for decimals, e.g. 0.3 x 4 • Multiply pairs of multiples of 10 with differing place value ,e.g. 4000 x 30	• Halve any number including 2dp • Divide whole numbers and decimals by 10,100,1000 • Use place value and known facts for decimals, e.g. 3.2 ÷ 8 • Divide pairs of multiples of 10 with differing place value, e.g. 8000 ÷ 200 • Divide by 50 by dividing by 100 and doubling • Divide by 25 by dividing by 100 and doubling and doubling again • Divide by 20 by dividing by 10 and halving • Divide by 5 by diving by 10 and doubling

Experience has shown us that longer, more complex written methods often go wrong through the **mental calculations** within them.

It is essential that pupils are taught these mental calculation skills.

Once pupils have mastered the relevant mental and written methods for their year group, it is advisable for them to **reason about which method suits a given calculation – what was the most efficient way of doing it!**

Please see our mental calculation policy for further detail to support these expectations.

Addition

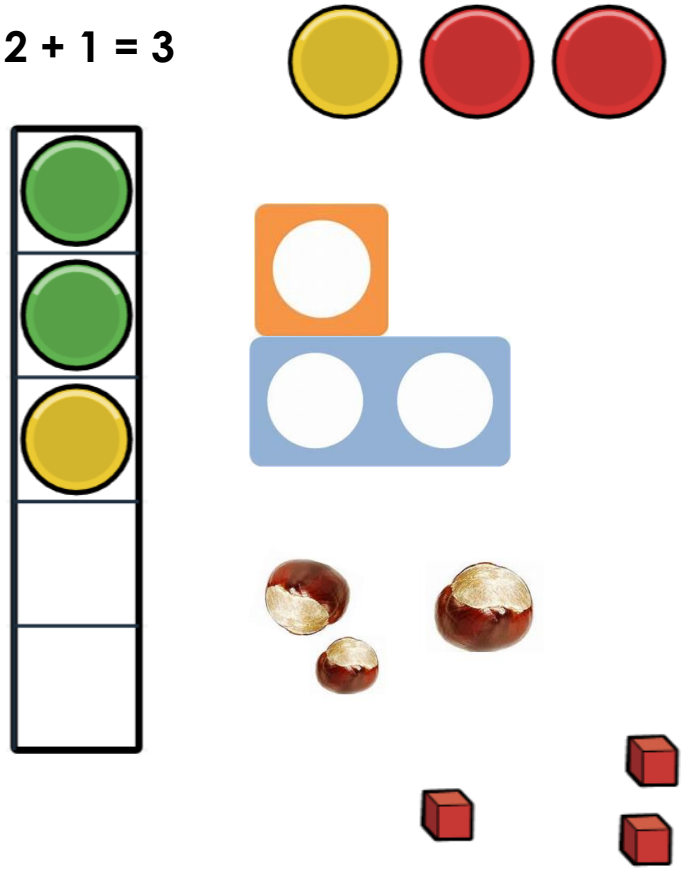
Trust Ready Document
Begin to add using practical resources to 3

Nursery

Concrete

Pupils to use a range of practical resources to add numbers up to three.

2 + 1 = 3



Pictorial

All addition work will fall within the concrete phase with practical resources at this age.

Abstract

All addition work will fall within the concrete phase with practical resources at this age.

ELG
Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

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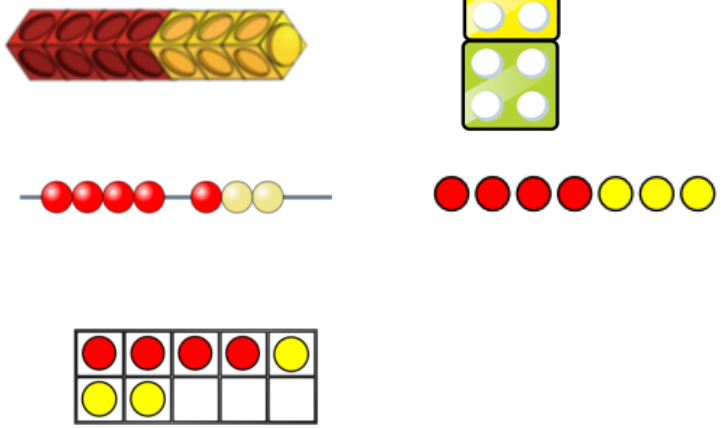
Reception

Concrete

Pupils to use a range of practical resources to add numbers up to ten. This must progress to using a tens frame.

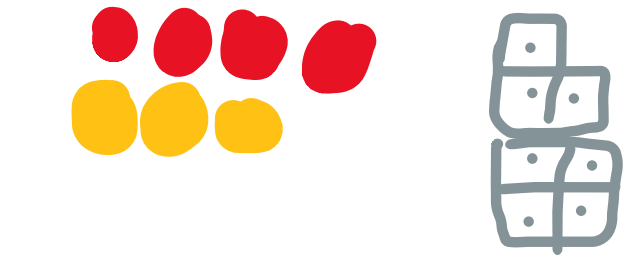
4 + 3 = 7

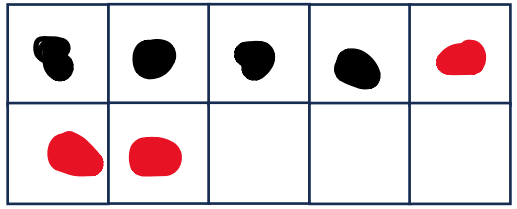


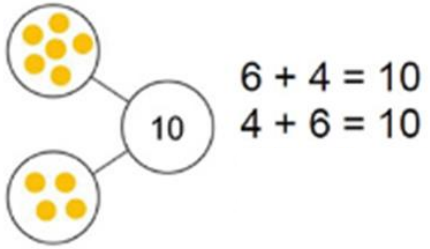
Pictorial

Pupils use simple diagrams, including mark making on prepared ten frames to calculate addition statements (number sentences).

4 + 3 = 7







6 + 4 = 10
4 + 6 = 10

Abstract

Pupils record the full statement and answer to a given addition statement (number sentence) to 10.

4 + 3 = 7

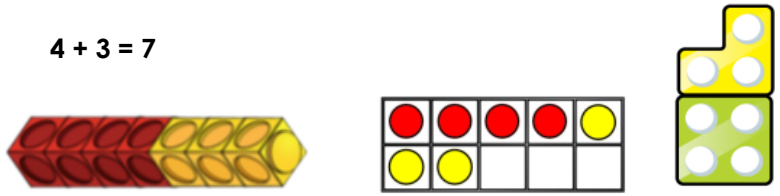
National Curriculum
add one-digit and two-digit numbers to 20, including zero; read, write and interpret mathematical statements involving addition (+) and equals (=) signs

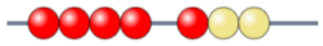
Year 1

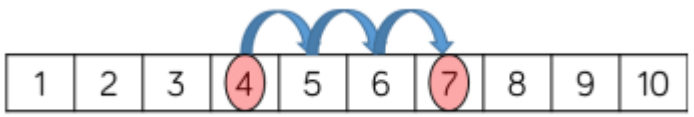
Concrete

Pupils to use a range of practical resources with calculations bridging through ten to <u>not</u> use exchange. The concept if ten ones equalling one ten though is to be emphasized. They should progress to using labelled, physical number lines.

4 + 3 = 7



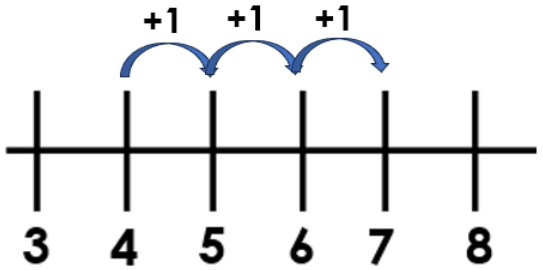




Pictorial

Pupils to use a printed, labelled number line to count in steps of one for addition.

4 + 3 = 7



Abstract

Pupils to record their addition calculations as mathematical statements (number sentences) using the addition and subtraction symbols.

4 + 3 = 7

National Curriculum
add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, two two-digit numbers adding three one-digit numbers

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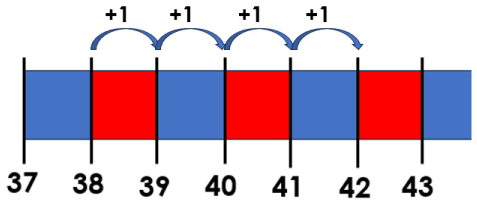
Year 2

Concrete

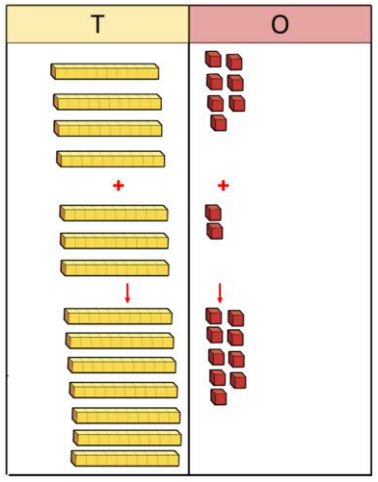
Pupils to use labelled, physical number lines to bridge tens.

38 + 4 = 42

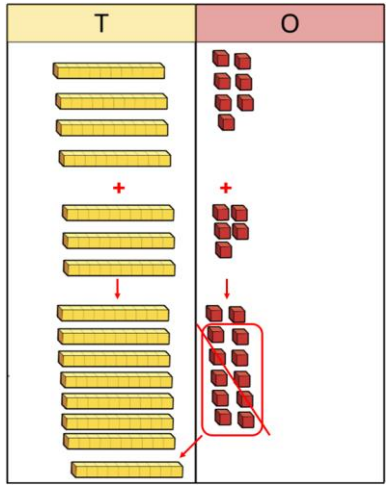


Pupils to use Base 10 to practically experience adding and regrouping. This must also be done with counters ready for Year 3.

47 + 32 = 79



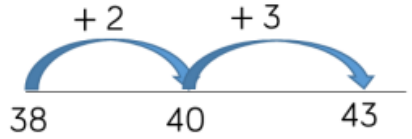
47 + 35 = 82



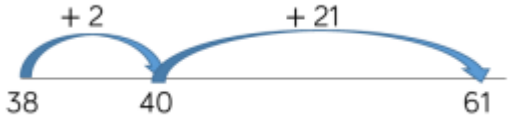
Pictorial

Pupils to draw their own blank number lines to bridge tens.

38 + 5 = 43

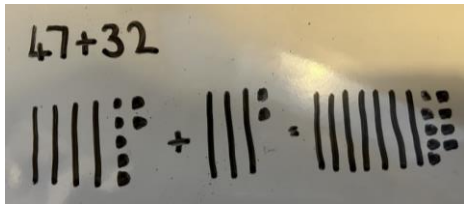


38 + 23 = 61

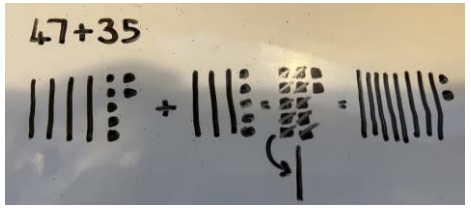


Pupils to draw Base 10 images, which again must move into using counters ready for Year 3.

47 + 32 = 79



47 + 35 = 82



Abstract

Pupils to use informal jottings to bridge through tens using the understanding developed using number lines.

38 + 4 = 43

38 + 2 = 40
40 + 2 = 42

38 + 23 = 61

38 + 2 = 40
40 + 21 = 63

Pupils to use expanded method with no regrouping before moving onto it.

47 + 32 = 79

	40	+	7		
+	30	+	2		
	70	+	9	=	79

47 + 35 = 82

	40	+	7		
+	30	+	5		
	70	+	12	=	82

National Curriculum
add numbers with up to three digits, using formal written methods of columnar addition

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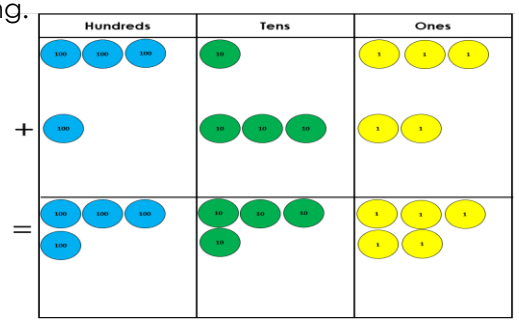
Year 3

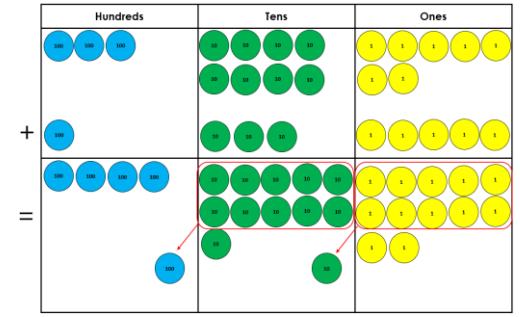
Concrete

Pupils to use counters (or **Base** 10)to practically experience adding.

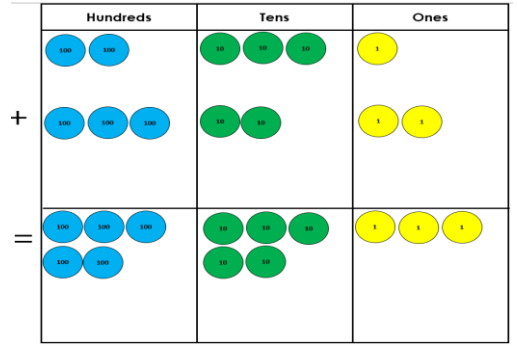
313 + 132 = 445



387 + 135 = 522



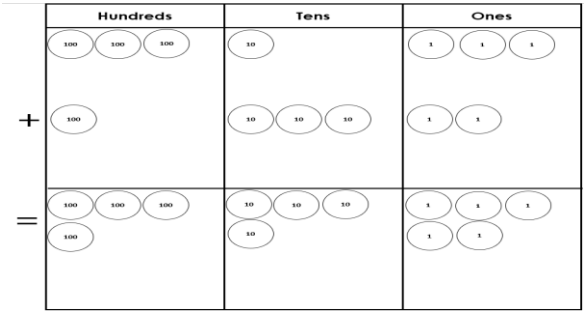
231 + 322 = 553



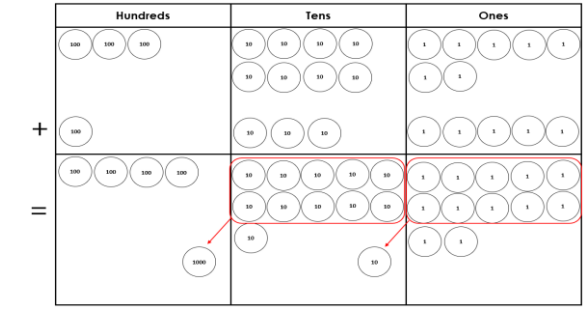
Pictorial

Pupils to draw counters without crossing out for regrouping.

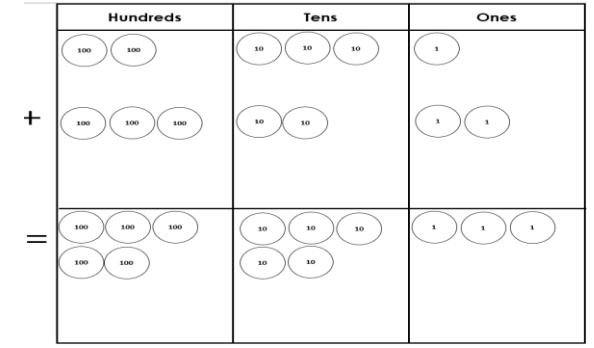
313 + 132 = 445



387 + 135 = 522



231 + 322 = 553



Abstract

Pupils to use expanded method with no regrouping before moving onto it.

$$\begin{array}{rrrrrrr} & 300 & + & 10 & + & 3 & \\ - & 100 & + & 30 & + & 2 & \\ \hline & 400 & & 40 & + & 5 & = 445 \end{array}$$

$$\begin{array}{rrrrrrr} & 300 & + & 80 & + & 7 & \\ + & 100 & + & 30 & + & 5 & \\ \hline & 400 & + & 110 & + & 12 & = 522 \end{array}$$

Pupils to move onto trying compact, standard method ready for Year 4 when secure with the above.

$$\begin{array}{rrrr} & 2 & 3 & 1 \\ + & 3 & 2 & 2 \\ \hline & 5 & 5 & 3 \end{array}$$

National Curriculum
add numbers with up to 4 digits using the formal written method of columnar addition where appropriate

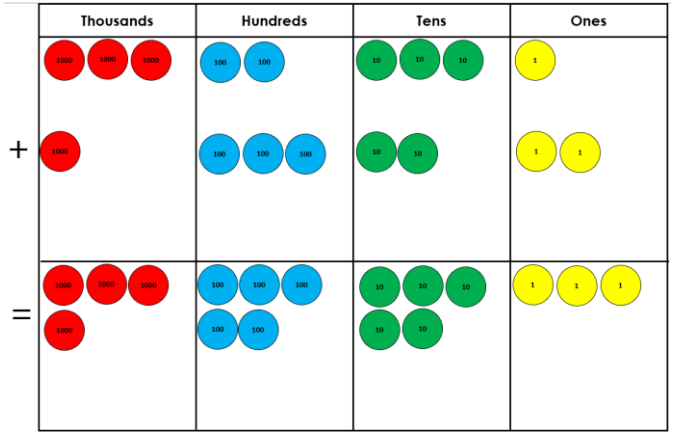
Lingfield Education Trust

Year 4

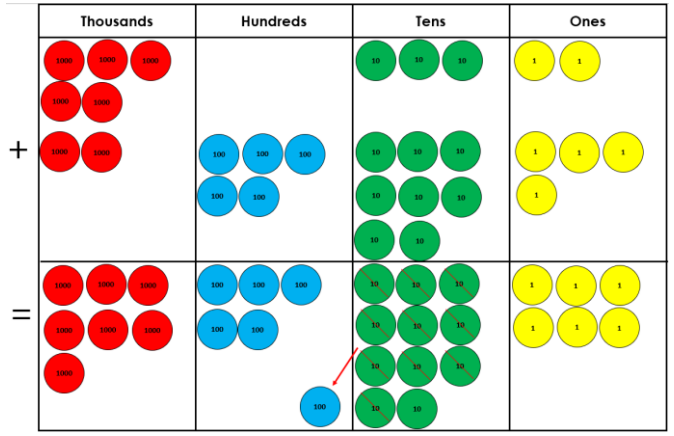
Concrete

Pupils to use counters to practically experience adding.

3231 + 1322 = 4553



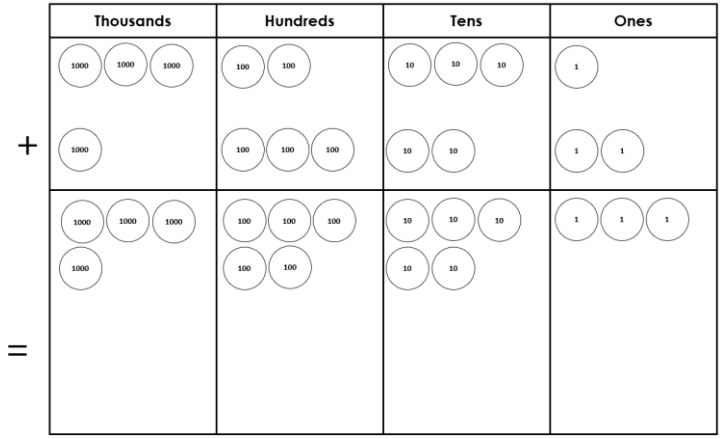
5032 + 2584 = 7616



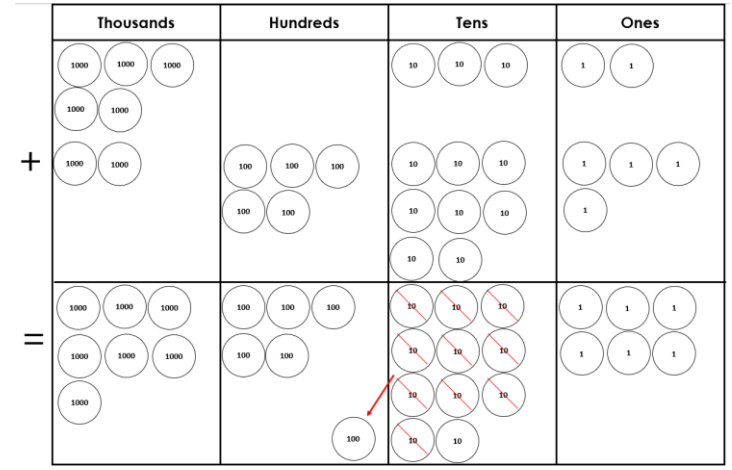
Pictorial

Pupils to draw counters crossing out for regrouping.

3231 + 1322 = 4553



5032 + 2584 = 7616



Abstract

Use of column method to add up to two 4-digit numbers (begin without regrouping and progress to regrouping).

$$\begin{array}{r} 3\ 2\ 3\ 1 \\ +\ 1\ 3\ 2\ 2 \\ \hline 4\ 5\ 5\ 3 \end{array}$$

$$\begin{array}{r} 5\ 0\ 3\ 2 \\ +\ 2\ 5\ 8\ 4 \\ \hline 7\ 6\ 1\ 6 \\ \hline 1 \end{array}$$

National Curriculum
add whole numbers with more than 4 digits, including using formal written method (columnar addition)

Year 5

Concrete

By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Pictorial

By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Abstract

Use of column addition for numbers including millions before using for numbers with up to three decimals places.

3,495,032 + 642,584 =

$$\begin{array}{r} 3\,4\,9\,5\,0\,3\,2 \\ +\,0\,6\,4\,2\,5\,8\,4 \\ \hline 4\,1\,3\,7\,6\,1\,6 \\ \hline \quad\;\; 1 \quad\quad\; 1 \end{array}$$

341.924 + 64.294 =

$$\begin{array}{r} 3\,4\,1\,.\,9\,2\,4 \\ +\quad 6\,4\,.\,2\,9\,4 \\ \hline 4\,0\,6\,.\,2\,1\,8 \\ \hline \quad 1 \quad\;\; 1 \quad\;\; 1 \end{array}$$

National Curriculum
solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Year 6

Concrete

By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Pictorial

By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Abstract

Use of column addition for numbers including millions before using for numbers with up to three decimals places.

3,495,032 + 642,584 =

	3	4	9	5	0	3	2
+	0	6	4	2	5	8	4
	4	1	3	7	6	1	6
		1			1		

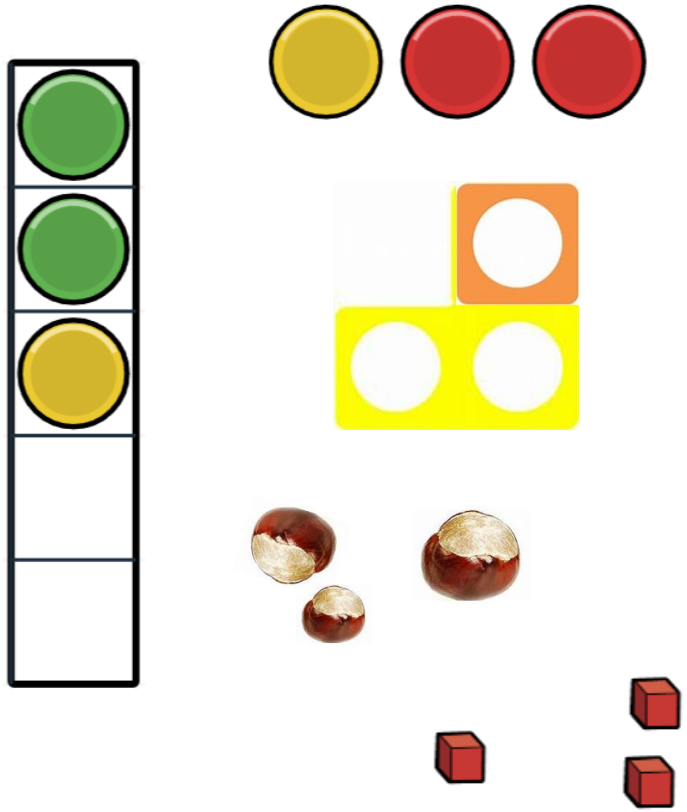
341.924 + 64.294 =

	3	4	1	.	9	2	4
+		6	4	.	2	9	4
	4	0	6	.	2	1	8
		1		1		1	

Subtraction

Trust Ready Document
Begin to subtract using practical resources to 3

Nursery

Concrete	Pictorial	Abstract
Pupils to use a range of practical resources to subtract numbers up to three. **3 - 1 = 2** 	All addition work will fall within the concrete phase with practical resources at this age.	All addition work will fall within the concrete phase with practical resources at this age.

ELG
Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

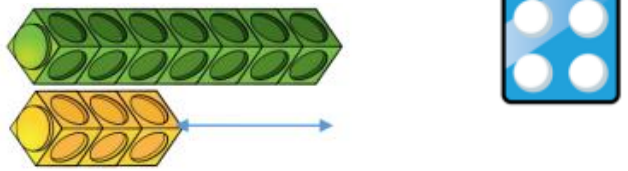
Lingfield Education Trust

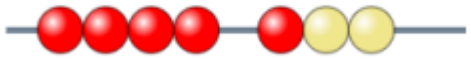
Reception

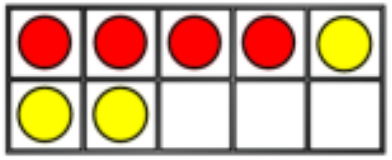
Concrete

Pupils to use a range of practical resources to subtract numbers up to ten. This must progress to using a tens frame.

7 - 3 = 4



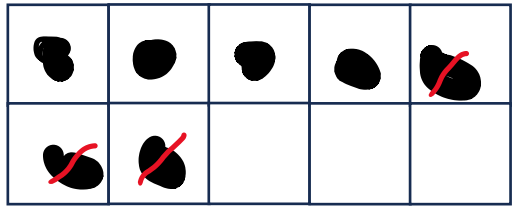


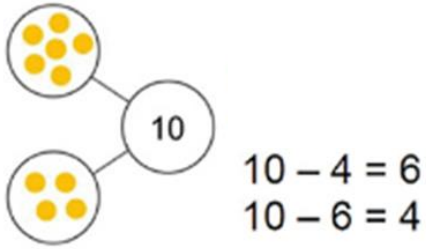


Pictorial

Pupils use simple diagrams, including mark making on prepared ten frames to calculate subtraction statements (number sentences).

7 - 3 = 4





Abstract

Pupils record the full statement and answer to a given subtraction statement (number sentence) to 10.

7 - 3 = 4

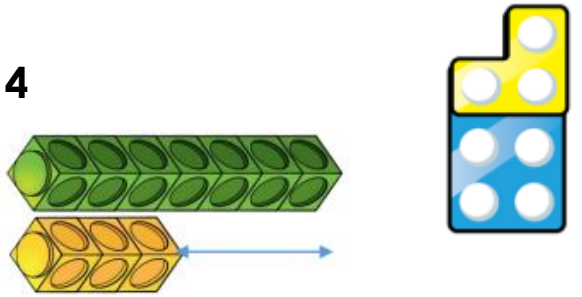
National Curriculum
subtract one-digit and two-digit numbers to 20, including zero; read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs

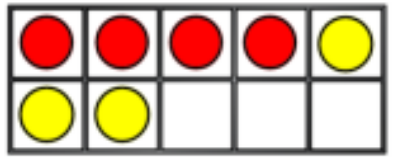
Year 1

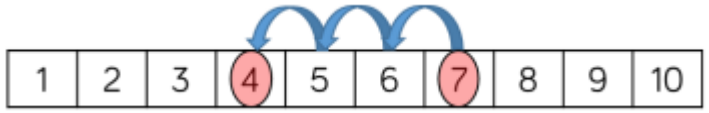
Concrete

Pupils to use a range of practical resources to subtract numbers up to twenty. This must progress to using physical number lines.

7 - 3 = 4



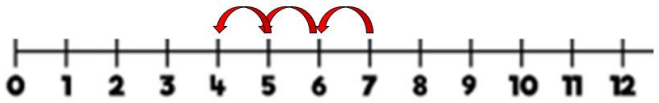




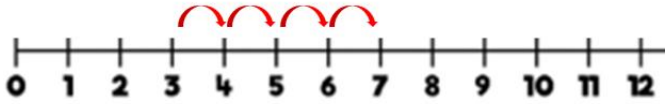
Pictorial

Pupils to use a printed number line to back in steps of one for the reduction structure of subtraction.

7 - 3 = 4



Pupils to use a printed number line to count on in steps of one for the comparative difference structure of subtraction.



Abstract

Pupils to record their subtraction calculations as mathematical statements (number sentences) using the addition and subtraction symbols.

7 – 3 = 4

National Curriculum
subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, two two-digit numbers adding three one-digit numbers

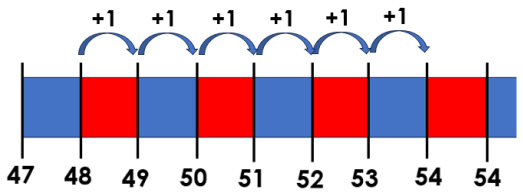
Lingfield Education Trust

Year 2

Concrete

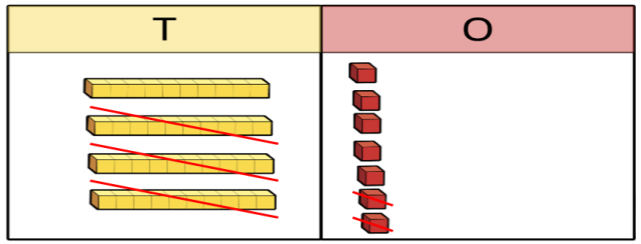
Pupils to use labelled, physical number lines to bridge tens.

54 - 6 = 48

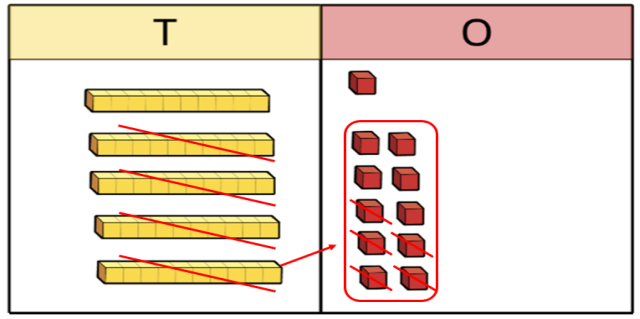


Pupils to use Base 10 to practically experience subtracting and regrouping. This must also be done with counters ready for Year 3.

47 - 32 = 15



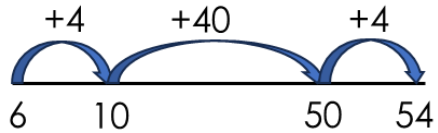
51 - 35 = 16



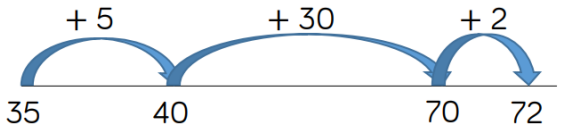
Pictorial

Pupils to draw their own blank number lines counting on to bridge tens.

54 - 6 = 48

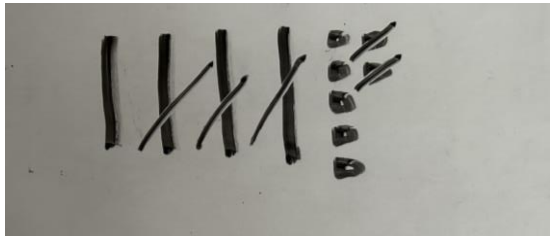


72 - 35 = 37

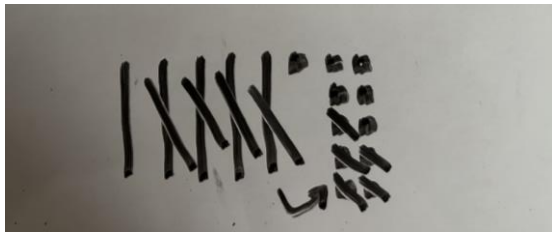


Pupils to draw Base 10 images, which again must move into using counters ready for Year 3.

47 - 32 = 15



51 - 35 = 16



Abstract

Pupils to use informal jottings to bridge through tens using the understanding developed using number lines.

54 - 6 = 48

54 - 4 = 50
50 - 2 = 48

72 - 35 = 37

72 - 30 = 42
42 - 2 – 3 = 37

Pupils to use expanded method with no regrouping before moving onto it.

47 - 32 = 15

	40	+	7		
-	30	+	2		
	10	+	5	=	15

51 - 35 = 16

	40 ~~50~~	+	11 ~~1~~		
-	30	+	5		
	10	+	6	=	16

National Curriculum
subtract numbers with up to three digits, using formal written methods of columnar subtraction

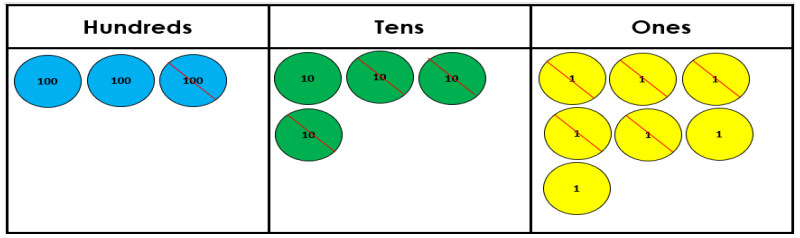
Lingfield Education Trust

Year 3

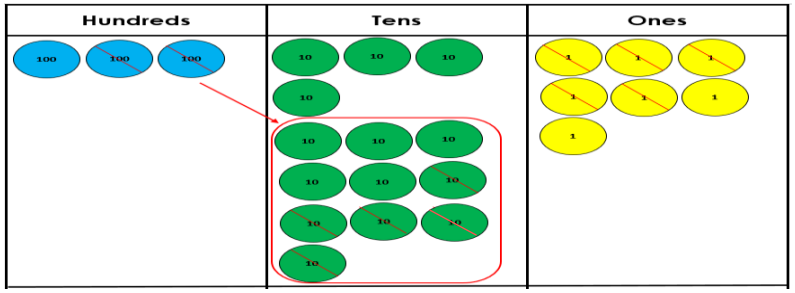
Concrete

Pupils to use Base 10 or counters to practically experience subtracting.

347 – 135 = 212

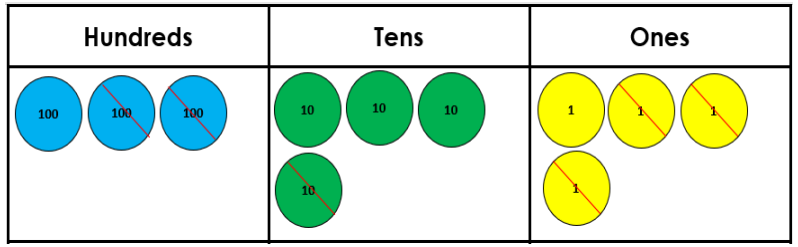


347 – 155 = 192



Pupils to use counters to practically experience subtracting.

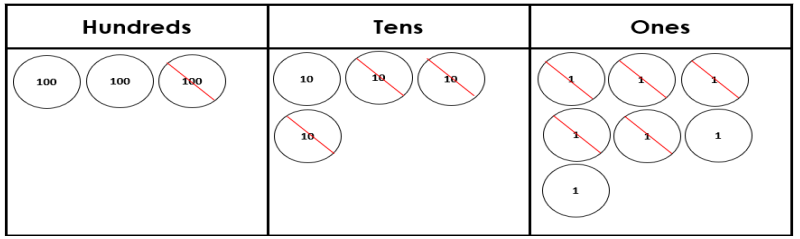
344 – 213 = 131



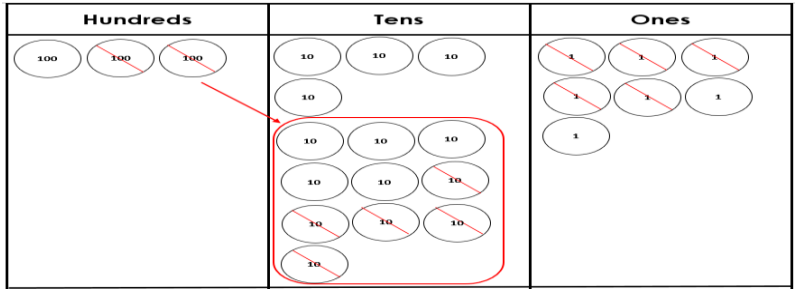
Pictorial

Pupils to draw Base 10 or counters with no regrouping before moving onto it via crossing out.

347 – 135 = 212

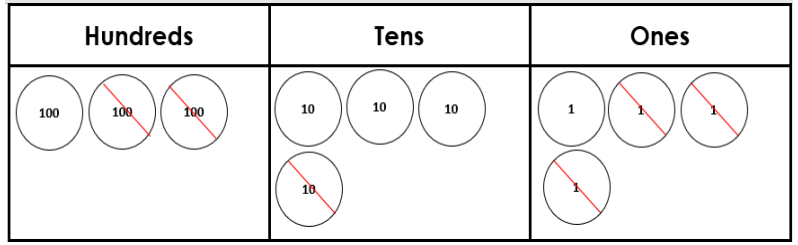


347 – 155 = 192



Pupils to draw counters crossing out for regrouping ready for Year 4.

344 – 213 = 131



Abstract

Pupils to use expanded method with no regrouping before moving onto it.

$$\begin{array}{rrrrrrr} & 300 & + & 40 & + & 7 & \\ - & 100 & + & 30 & + & 5 & \\ \hline & 200 & + & 10 & + & 2 & = 212 \end{array}$$

$$\begin{array}{rrrrrrr} & \overset{200}{\cancel{3}00} & + & \overset{140}{\cancel{4}0} & + & 7 & \\ - & 100 & + & 50 & + & 5 & \\ \hline & 100 & + & 90 & + & 2 & = 192 \end{array}$$

Pupils to move onto trying compact, standard method ready for Year 4 when secure with the above.

$$\begin{array}{rrrr} & 3 & 4 & 4 \\ - & 2 & 1 & 3 \\ \hline & 1 & 3 & 1 \end{array}$$

National Curriculum
subtract numbers with up to 4 digits using the formal written method of columnar subtraction where appropriate

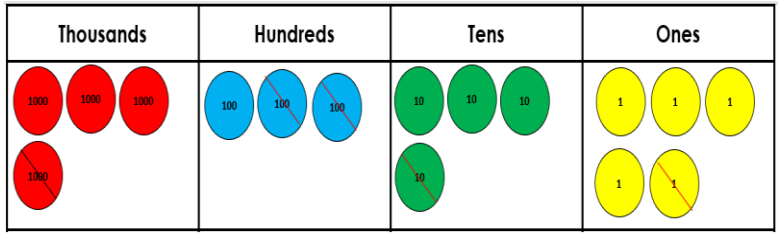
Lingfield Education Trust

Year 4

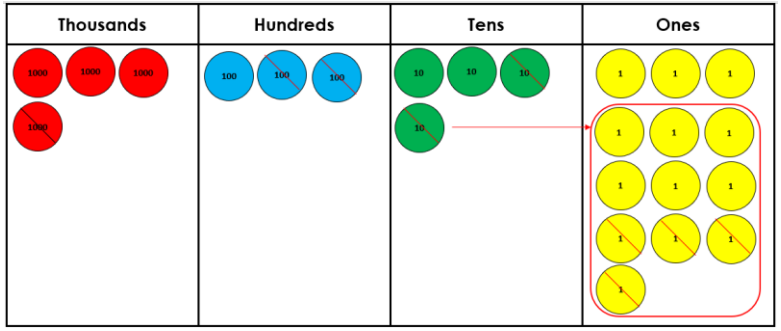
Concrete

Pupils to use counters to practically experience subtracting.

4345 – 1212 = 3133



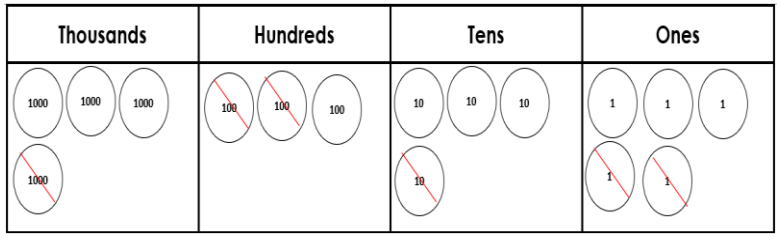
4343 – 1214 = 3129



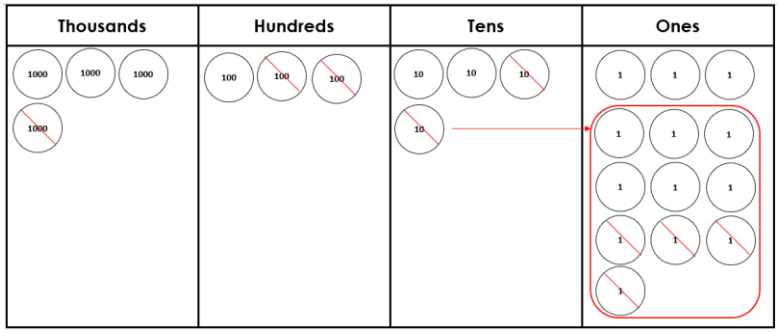
Pictorial

Pupils to draw counters crossing out for regrouping.

4345 – 1212 = 3133



4343 – 1214 = 3129



Abstract

Use of column method to subtract up to two 4-digit numbers (begin without regrouping and progress to regrouping).

$$\begin{array}{r} 4\ 3\ 4\ 5 \\ -\ 1\ 2\ 1\ 2 \\ \hline 3\ 1\ 3\ 3 \end{array}$$

$$\begin{array}{r} 4\ 3\ \overset{3}{\cancel{4}}\ \overset{1}{3} \\ -\ 1\ 2\ 1\ 4 \\ \hline 3\ 1\ 2\ 9 \end{array}$$

National Curriculum
subtract whole numbers with more than 4 digits, including using formal written method (columnar subtraction)

Year 5

Concrete

By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Pictorial

By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Abstract

Use of column subtraction for numbers including millions before using for numbers with up to three decimals places.

$$\begin{array}{r} {}^{2}\cancel{3}\ {}^{13}\cancel{4}\ {}^{1}1\ {}^{8}\cancel{9}\ {}^{11}\cancel{2}\ {}^{1}0 \\ -\quad 6\ 4\ 2\ 9\ 4 \\ \hline 2\ 7\ 7\ 6\ 2\ 6 \end{array}$$

$$\begin{array}{r} {}^{2}\cancel{3}\ {}^{13}\cancel{4}\ {}^{1}1\ .\ {}^{8}\cancel{9}\ {}^{11}\cancel{2}\ {}^{1}0 \\ -\quad 6\ 4\ .\ 2\ 9\ 4 \\ \hline 2\ 7\ 7\ .\ 6\ 2\ 6 \end{array}$$

National Curriculum
solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Year 6

Concrete

By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Pictorial

By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Abstract

Use of column subtraction for numbers including millions before using for numbers with up to three decimals places.

$$\begin{array}{r} {}^{2}\cancel{3}\ {}^{13}\cancel{4}\ {}^{1}1\ {}^{8}\cancel{9}\ {}^{11}\cancel{2}\ {}^{1}0 \\ -\quad 6\ 4\ 2\ 9\ 4 \\ \hline 2\ 7\ 7\ 6\ 2\ 6 \end{array}$$

$$\begin{array}{r} {}^{2}\cancel{3}\ {}^{13}\cancel{4}\ {}^{1}1\ .\ {}^{8}\cancel{9}\ {}^{11}\cancel{2}\ {}^{1}0 \\ -\quad 6\ 4\ .\ 2\ 9\ 4 \\ \hline 2\ 7\ 7\ .\ 6\ 2\ 6 \end{array}$$

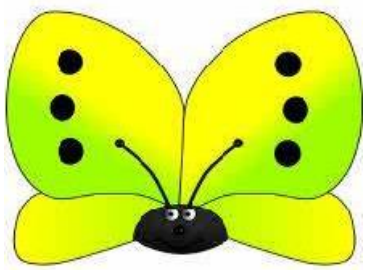
Multiplication

Trust Ready Document
Know that doubling and halving are related (inverse operation).

Reception

Concrete

Children use physical resources to solve multiplication problems involving doubling.

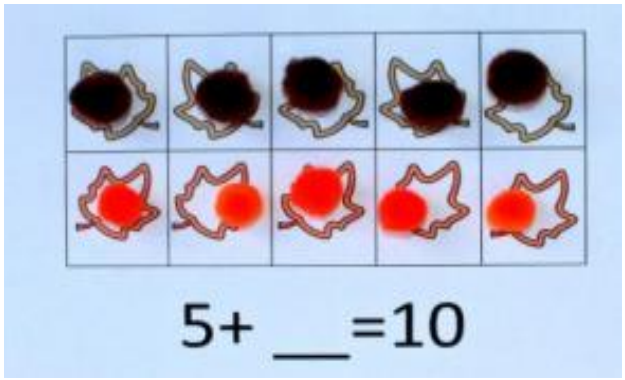


Double 3 is 6

3 + 3 = 6

Pictorial

Children use pictorial representations to solve multiplication problems involving doubling.



Abstract

All multiplication work will fall within the concrete and pictorial phase with practical resources at this age.

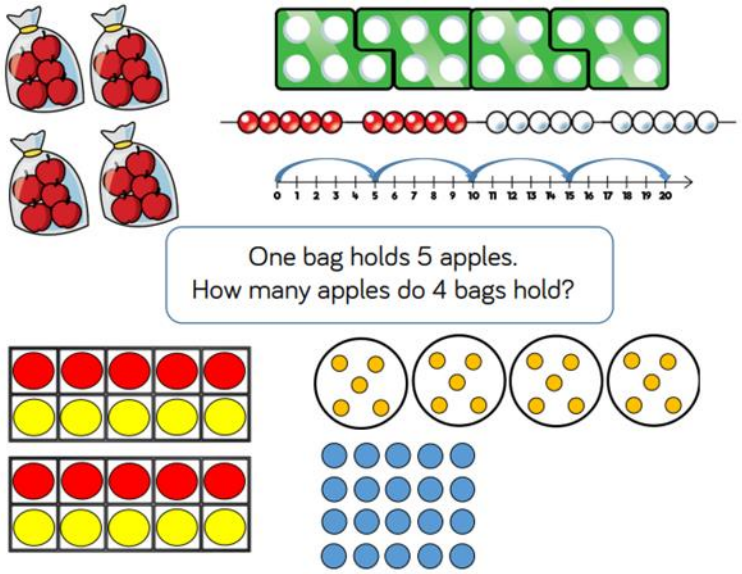
National Curriculum
solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Lingfield Education Trust

Year 1

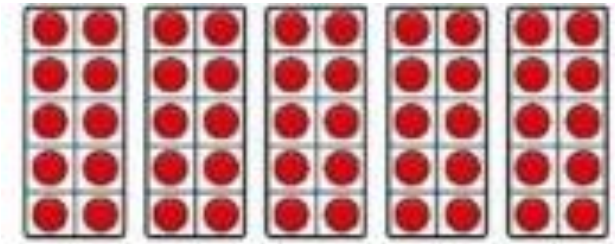
Concrete

In Year 1, children use concrete resources to solve multiplication problems. Children represent multiplication as repeated addition in many different ways. This should include physical, labelled number tracks ready for the pictorial phase.

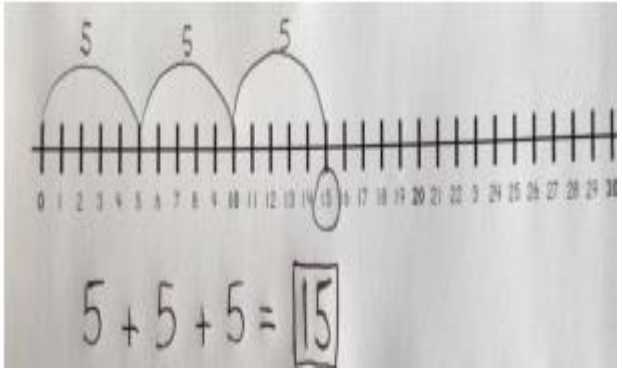


Pictorial

Use pictorial arrays to build understanding of multiplication through counting the total in amounts in 2s, 10s and 5s..



Use a number line to jump in multiples of 2, 5 and 10 (repeated addition).



Abstract

Use mathematical statements (number sentences) for repeated addition of 2, 5 or 10.

$$5 + 5 + 5 = 15$$

Introduce the multiplication symbol to replace repeated addition.

$$5 + 5 + 5 = 15$$

$$3 \times 5 = 15$$

National Curriculum
calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs

Year 2

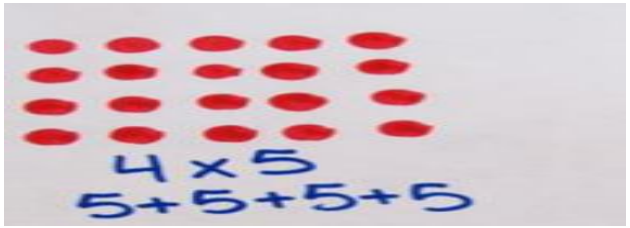
Concrete

Use a range of physical resources to practically experience repeated addition and multiplication.

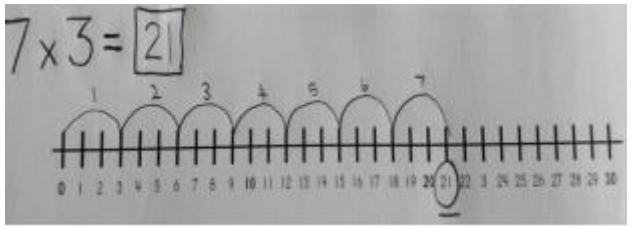
Numicon number tracks used alongside cuisenaire rods are an excellent way to bridge towards the use of number lines for repeated addition.

Pictorial

Make marks to create arrays show repeated addition of 2, 3, 5 or 10.



Use a number line to represent jumps in groups of 2, 3, 5 and 10 (counting on using repeated addition) where the number of jumps will equal the number of groups.



Children can progress to drawing their own number lines.

Abstract

Write repeated addition sentences to match sets of objects or pictures.

5 + 5 + 5 + 5 = 20

Use the multiplication symbol to replace repeated addition.

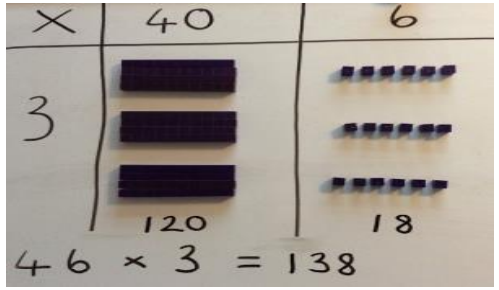
7 x 3 = 21

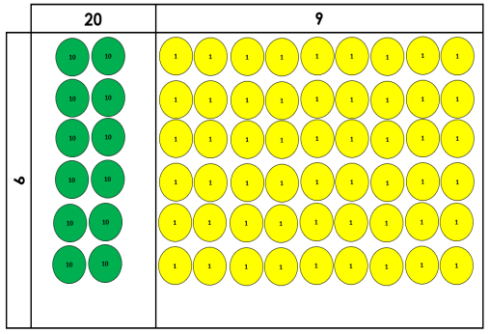
National Curriculum
write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Year 3

Concrete

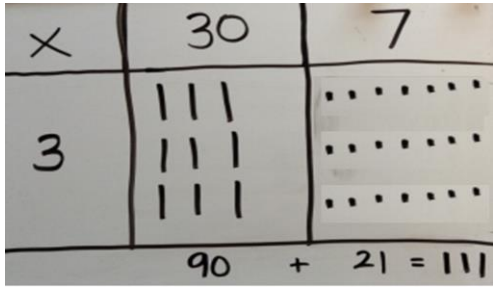
Create arrays using dienes and position these correctly on a grid (to introduce the grid method for 2-digit x 1-digit). Progress to use counters ready for Year 4.

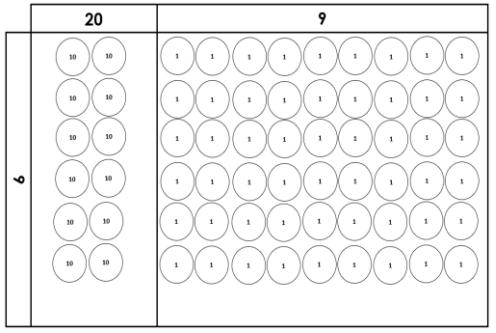




Pictorial

Use marks to represent Base10 on a multiplication grid method (2-digit x 1-digit) and likewise for counters.





Abstract

Replace resources/marks with digits on an expanded grid.

x	20	9
6	20	9
	20	9
	20	9
	20	9
	20	9
	20	9

120 + 54 = 174

Move on to the grid method.

x	20	9
6	120	54

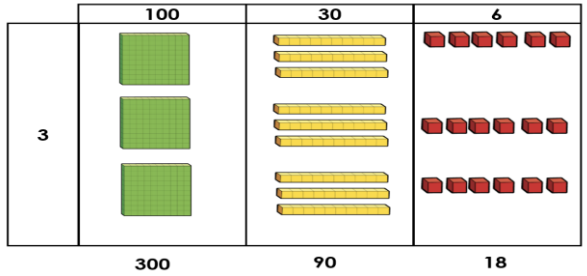
120 + 54 = 174

National Curriculum
multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Year 4

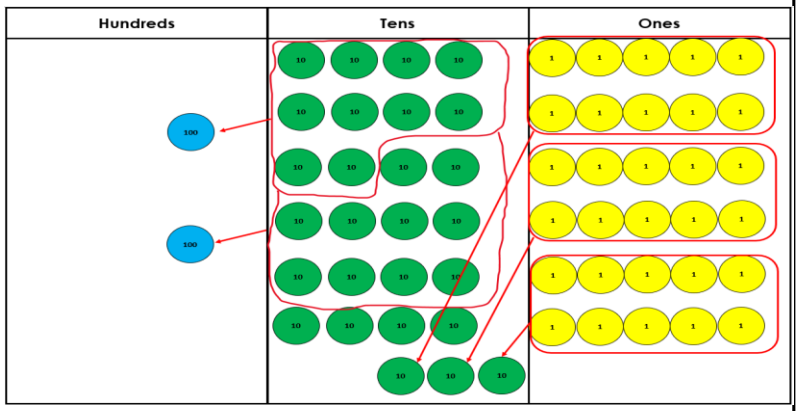
Concrete

Pupils to use Base 10 to support multiplication.



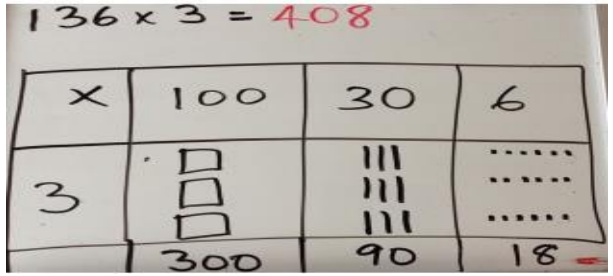
Pupils to use counters to support multiplication using regrouping as they do.

45 x 6



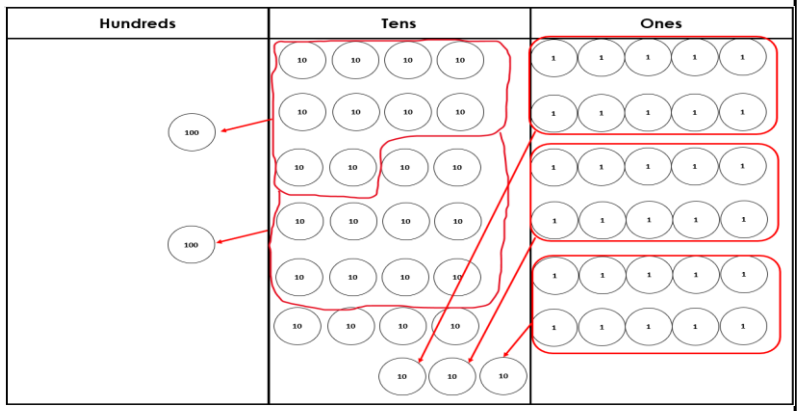
Pictorial

Pupils to create pictorial representations of Base 10 to support multiplication.



Pupils to draw counters to support multiplication using regrouping as they do.

45 x 6



Abstract

Introduce short multiplication as a formal written method for multiplying 2 or 3 digit numbers by 1 digit numbers using the expanded method to show the addition of two products.

	324	
x	2	
	8	2 x 4
	40	2 x 20
	600	2 x 300
	648	

Use of short multiplication for multiplying 3-digit by 1-digit numbers.

		4	5
x			6
	2	7	0
		3	

		3	4	5
x				6
	2	0	7	0
		2	3	

National Curriculum
multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

Lingfield Education Trust

Year 5

Concrete

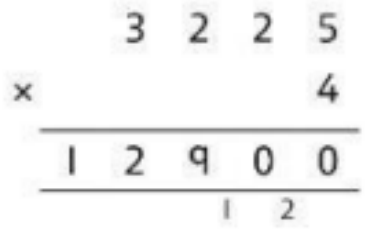
By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Pictorial

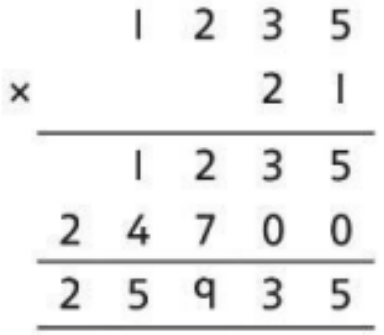
By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Abstract

Use of short multiplication for multiplying 4-digit by 1-digit numbers.



Use of long multiplication for multiplying 4-digit by 2-digit numbers.



National Curriculum
multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Lingfield Education Trust

Year 6

Concrete

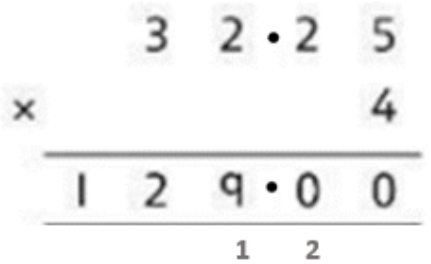
By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

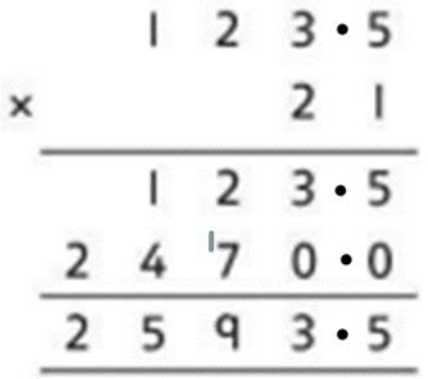
Pictorial

By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Abstract

Consolidate use of short and long multiplication with integers from Year 5, before using for decimals with up to two decimal places.





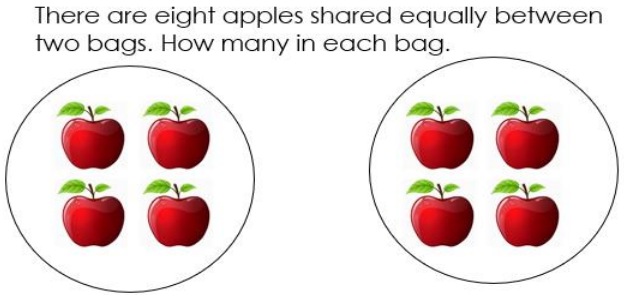
Division

Trust Ready Document
Know that doubling and halving are related (inverse operation).

Reception

Concrete

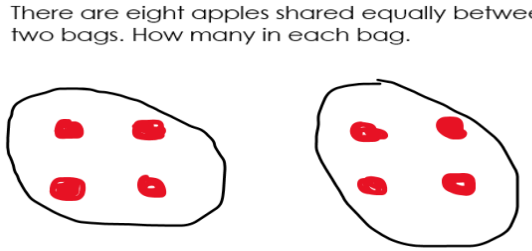
Children solve division problems by **sharing** amounts into two equal groups to develop concept of halving. Children use concrete resources to solve problems.



Children also solve problems by **grouping** and counting the number of groups.

Pictorial

Children solve division problems by sharing amounts into equal groups. Children use pictorial representations to solve problems involving two groups and halving.



Children also solve problems by **grouping** and counting the number of groups using pictorial representations, including number lines ready for Year 1.

Abstract

All division work will fall within the concrete and pictorial phase with practical resources at this age.

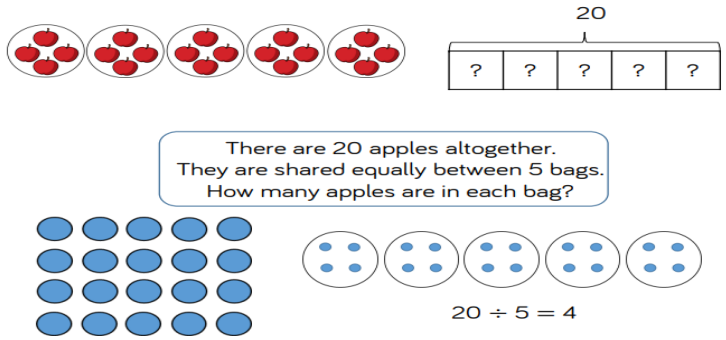
National Curriculum
solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Lingfield Education Trust

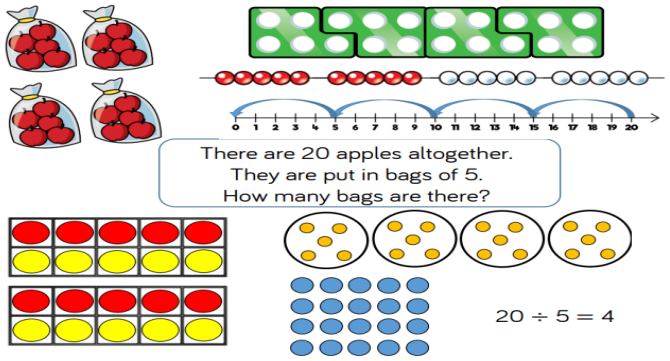
Year 1

Concrete

Children solve division problems by **sharing** amounts into equal groups. Children use concrete resources to solve problems.

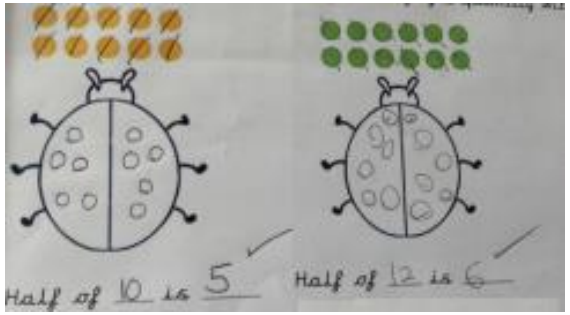


Children also solve problems by **grouping** and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line ready for Year 2.

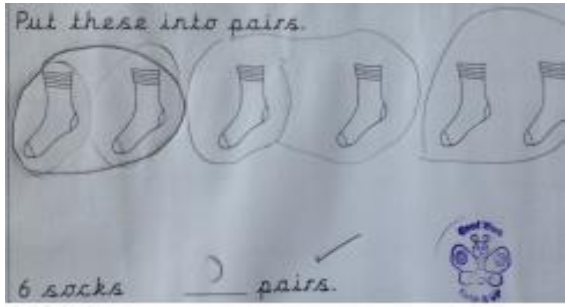


Pictorial

Children solve division problems by sharing amounts into equal groups. Children use pictorial representations to solve problems.



Children also solve problems by **grouping** and counting the number of groups using pictorial representations, including number lines ready for Year 2.



Abstract

Introduce the division symbol to record sharing calculations.

$$20 \div 5 = 4$$

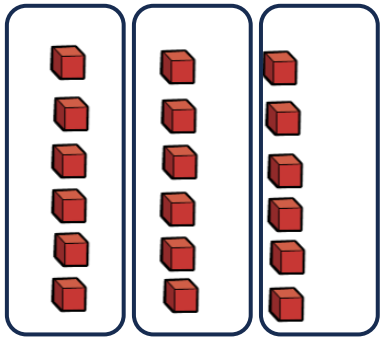
National Curriculum
calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs

Year 2

Concrete

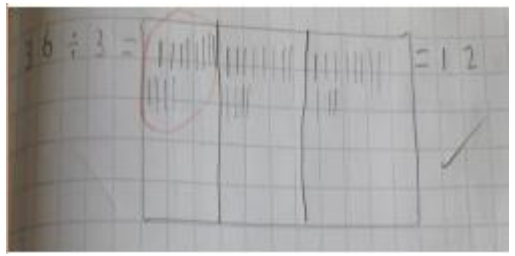
Use of concrete apparatus for sharing and grouping to continue.

18 ÷ 3 = 6

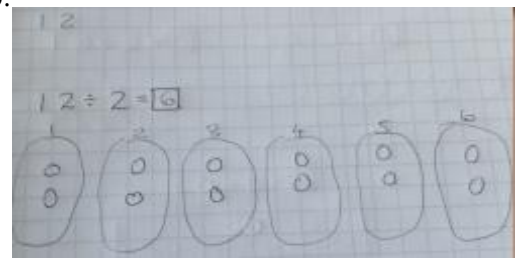


Pictorial

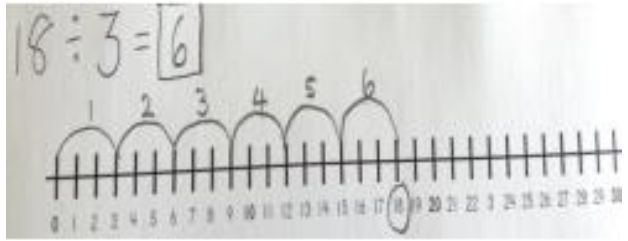
Children make marks to show sharing between 2, 3, 5, or 10.



Children make marks to show division by grouping sets of 2, 3, 5, or 10.



Progress to use of a number line to represent jumps in groups of 2, 3, 5 and 10 (counting on using repeated addition) where the number of jumps will equal the number of groups.



Abstract

Pupils to write their own division statements to record their calculations using the division and equals symbols..

18 ÷ 3 = 6

National Curriculum
write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers by one-digit numbers, using mental and progressing to formal written methods

Year 3

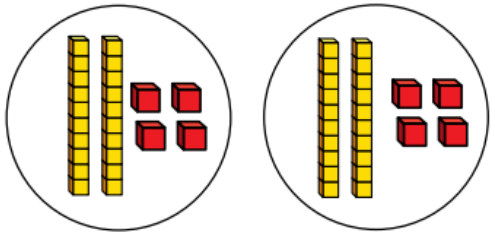
Concrete

Use of concrete apparatus for sharing and grouping to continue.

When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

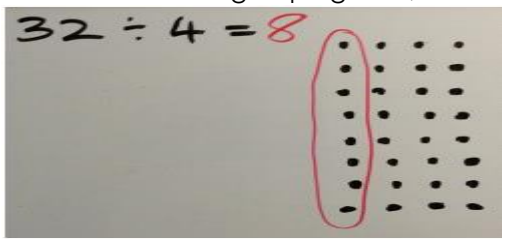
48 ÷ 2

Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1

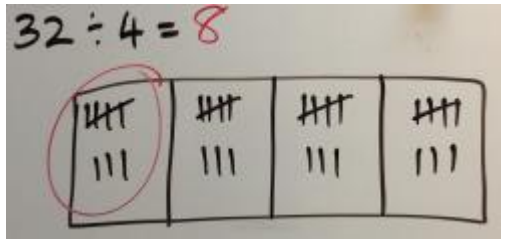


Pictorial

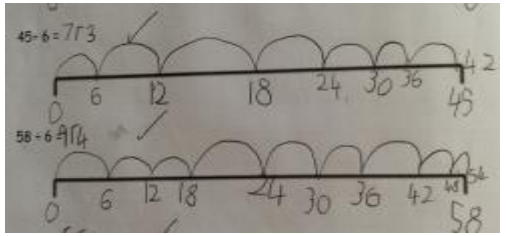
Pupils use marks to show grouping in 4s, 6s and 8s.



Use marks to show sharing in 4s, 6s and 8s.



Use a number line to represent jumps in groups of 2, 3, 4, 5, 6, 8 and 10 (counting on using repeated addition) where the number of jumps will equal the number of groups and the number left over is the remainder.



Abstract

Use the division symbol to record calculations when dividing by 2, 3, 4, 5, 6, 8 and 10. Make explicit links between multiplication and division.

$36 \div 3 = 12$ $36 \div 12 = 3$

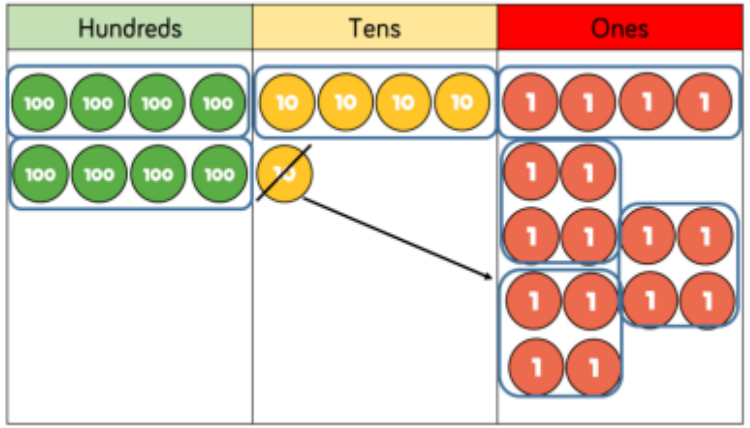
National Curriculum
Pupils practise to become fluent in the formal written method of short division with exact answers

Year 4

Concrete

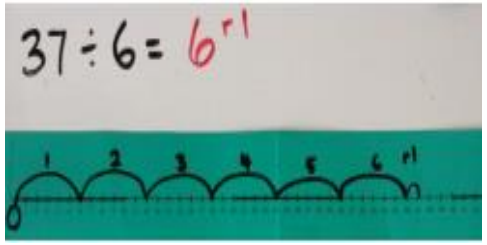
Children can continue to use grouping to support their understanding of short division when dividing a 2 or 3-digit number by a 1-digit number.

856 ÷ 4 = 214

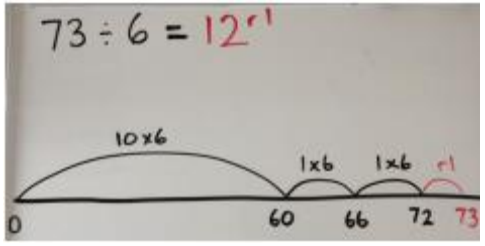


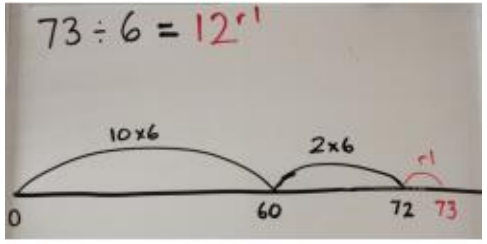
Pictorial

Use a number line to represent jumps in equal groups using all multiplication facts (as in year 3 – repeated addition) if required. This is consolidation and linking to Year 3.



Use a number line to count 'ten lots of' / 'ten groups of' and find remainders (chunking method). Progress to children choosing their own way of chunking using known multiplication facts.





Abstract

Do not use the flexible strategy in White Rose for main written method – that can be used as a mental strategy.

Use of short division for dividing 2-digit numbers by 1-digit numbers (links to the number line work) with no remainders and then remainders.

$$\begin{array}{r} 2\ 1 \\ 4\overline{)\,8\ 4} \end{array}$$

$$\begin{array}{r} 2\ 2 \\ 4\overline{)\,8\ 9} \end{array} \quad r1$$

Progress to use of short division for dividing 3-digit numbers by 1-digit numbers (links to the number line work) with no remainders and then remainders.

$$\begin{array}{r} 1\ 2\ 2 \\ 4\overline{)\,4\ 8\ 8} \end{array}$$

$$\begin{array}{r} 1\ 2\ 1 \\ 4\overline{)\,4\ 8\ 7} \end{array} \quad r3$$

National Curriculum
divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Year 5

Concrete

By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

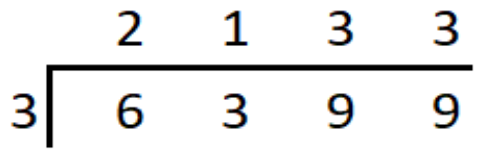
Pictorial

By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

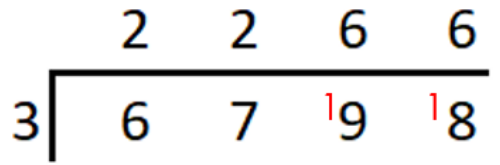
Abstract

Use short division for up to 4-digit numbers divided by a single digit including remainders .

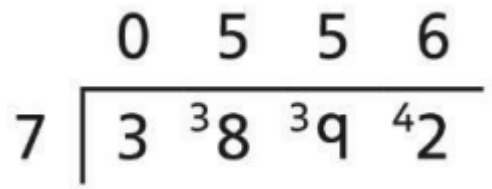
Start with exact answers throughout.



Progress to no remainders but regrouping within.



Progress to remainders and regrouping within.



National Curriculum
divide numbers up to 4 digits by a two-digit whole number using the formal written method of long (or short) division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate

Year 6

Concrete

By Year 6 pupils should be competent with using the abstract method only. Use of the concrete stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Pictorial

By Year 6 pupils should be competent with using the abstract method only. Use of the pictorial stage from prior year groups can be used for intervention with pupils working below age-related expectations.

Abstract

Consolidate short division from Year 5 before introducing long division for 4-digit numbers by 2-digit numbers (you may wish to start with 3-digit by 2-digit).

```
        0  2  1  2
    ┌──────────────
3 2 │ 6  7  8  4
      6  4
    ──────────────
         3  8
         3  2
    ──────────────
            6  4
            6  4
    ──────────────
            0  0
```

1 x 32 = 32
2 x 32 = 64
3 x 32 = 96
4 x 32 = 128
5 x 32 = 160
10 x 32 = 320

Move onto dividing 4-digit by 2-digit with remainders.

```
        0  2  1  2   r3
    ┌──────────────
3 2 │ 6  7  8  7
      6  4
    ──────────────
         3  8
         3  2
    ──────────────
            6  7
            6  4
    ──────────────
            0  3
```

1 x 32 = 32
2 x 32 = 64
3 x 32 = 96
4 x 32 = 128
5 x 32 = 160
10 x 32 = 320

Look at efficiency and when short division can be used for 4-digit by 2-digit.

```
        0  4  8  9
    ┌──────────────
1 5 │ 7  3  3  5
       7  13 13
```

1 x 15 = 15
2 x 15 = 30
3 x 15 = 45
4 x 15 = 60
5 x 15 = 75
8 x 15 = 120

Acknowledgements

Resources Used

- White Rose Calculation Policy
- Captain Cook Primary School Calculation Policy
- NPCAT Calculation Policy
- Purposeful Maths Calculation Policy
- LET EYFS Ready Documents

Many thanks to the following contributors

K Yates (Hurworth), P Mayes (Corporation Road), P Foulds (Mount Pleasant), C Harrison (Captain Cook), A Poulter (Vane Road), S Guest (Cambrai), R Farrier (Handale), R Carass (Heathfield)